

Role of the nuclear vector potential in deep inelastic scattering

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Abstract

We study the influence of the strong nuclear vector potential, treated using the mean-field approximation, in deep inelastic scattering. A consistent treatment of the electromagnetic current operator, combined with the use of the operator product expansion is presented and discussed.

I. INTRODUCTION

The EMC effect revealed that the structure of nucleons bound within the nucleus differs from those of free space [1, 2]. Indeed, careful treatments [3]-[7] of Fermi motion and binding effects of nucleons cannot explain the observed reduction of the nuclear structure function, or distribution function $q(x)$, in the range of Bjorken x between 0.3 and 0.8. Therefore it is worthwhile to derive models of nuclei in which the internal quark structure of nucleons responds to the nuclear environment. One may then investigate whether such responses account for the observations.

This is a daunting task. One way to proceed [8]-[17] is to use a mean field models in which quarks in a given nucleon feel the influence of mesons produced by the average field of the nucleus. Such models should account for the saturation properties of nuclear matter, as well as the structure functions of the free nucleon. Early work on this problem [8] was based on the quark-meson coupling model [9] in which quarks interact by the exchange of scalar and vector mesons. More recently, a quark-diquark description of the single nucleon [13, 14], based on the NJL model [10], was combined with the mean field description of nuclear matter [11, 12]. Another set of work is based on a nuclear matter version [17] of the chiral quark soliton model of Diakonov *et al.* [18]. Here, the nuclear attraction is generated by the exchange of pairs of pions (yielding an effective scalar potential) with the environment, and the repulsion arises from vector meson exchange.

A general feature of these mean-field models is that the binding interactions can be expressed in terms of scalar and vector potentials. The treatment of the scalar potentials is straightforward, but the vector potential is more subtle. In present mean-field models the vector potential is a constant that changes the energy of a nucleon, but causes no change in the wave function. It has been argued that the quark struck by the hard photon should not feel the nucleon vector potential $3V^0$, and accounting for this causes a shift in the argument of the distribution function, and a change in its normalization. This shift seems to have a significant effect on computed numerical results [8, 16], but its presence is not immediately apparent in other work [17].

The sole aim of the present note is to clear up the technical issue of whether it is necessary, if one is using a mean field approximation, to include this shift in the argument. To focus on the main point we simplify and use infinite nuclear matter (in which the scalar and vector potentials are treated as constants), examine only the spin-averaged structure function, and ignore QCD radiative effects by assuming the Bjorken limit.

We proceed by discussing the standard derivation of the parton model [3, 19] for a free nucleon, and then immerse the nucleon in the medium. The paper ends by providing an interpretation that unifies the approaches [8, 16] and [17].

II. FREE NUCLEON

Consider charged lepton scattering on a free nucleon in which the initial lepton exchanges a photon of momentum q with a target of momentum P . The differential cross-section for inclusive scattering depends on the hadronic tensor $W^{\mu\nu}$:

$$4\pi W^{\mu\nu} = \int d^4\xi e^{iq\cdot\xi} \langle PS | [J^\mu(\xi), J^\nu(0)] | PS \rangle_c, \quad (1)$$

expressed in terms of connected (c) matrix elements of the electromagnetic current operator J^μ :

$$J_\mu(\xi) = \bar{\psi}(\xi)\gamma_\mu\hat{Q}\psi(\xi). \quad (2)$$

The charge operator is \hat{Q} and the states are covariantly normalized to: $\langle P|P'\rangle = 2E(2\pi)^3\delta^3(P-P')$. Using Lorentz covariance, gauge invariance, parity conservation in electromagnetism and standard discrete symmetries of the strong interactions, $W^{\mu\nu}$ can be parametrized in terms of four scalar dimensionless structure functions $F_1(x, Q^2)$, $F_2(x, Q^2)$, $g_1(x, Q^2)$ and $g_2(x, Q^2)$. We shall be concerned with spin averaged quantities, and keep only the symmetric part of $W^{\mu\nu}$:

$$W_s^{\mu\nu} \equiv \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1 + \left[\left(P^\mu - \frac{\nu}{q^2}q^\mu\right)\left(P^\nu - \frac{\nu}{q^2}q^\nu\right)\right] \frac{F_2}{\nu}, \quad (3)$$

We use Jaffe's conventions [19] in which $W^{\mu\nu}$ is dimensionless.

We evaluate $F_1(x) = W_s^{11}$ in the parton model for a free nucleon using the laboratory system in which $q^\mu = \{q_0, 0_\perp, -\sqrt{q_0^2 + Q^2}\}$ and $P^\mu = \{M, 0_\perp, 0\}$. Using light-cone coordinates, $q \cdot \xi = q^+\xi^- + q^-\xi^+$, with $q^+ = -Mx/\sqrt{2}$ and $q^- = 2q^0 + \frac{Q^2}{2q^0}$ in the Bjorken limit.

The leading contribution in the operator product expansion of the cross section (or forward Compton amplitude) is the handbag diagram, obtained by evaluating the current commutator and keeping the most singular terms [19]. The identity

$$[\bar{\psi}_1\psi_1, \bar{\psi}_2\psi_2] = \bar{\psi}_1\{\psi_1, \bar{\psi}_2\}\psi_2 - \bar{\psi}_2\{\psi_2, \bar{\psi}_1\}\psi_1 \quad (4)$$

is useful. This is obtained by neglecting the (unequal-time) anti-commutator of the ψ fields, and is valid for non-interacting quarks or for quarks immersed in a constant background field. For a massless field,

$$\{\psi(\xi), \bar{\psi}(0)\} = \frac{1}{2\pi}\not{\partial}\epsilon(\xi_0)\delta(\xi^2), \quad (5)$$

(with $\epsilon(\xi_0) = 1$ for $\xi_0 \geq 0$ and $\epsilon(\xi_0) = -1$ for $\xi_0 < 0$). The relevant current commutator is then given by

$$\begin{aligned} [J^1(\xi), J^1(0)] &= -\frac{1}{2\pi}[\bar{\psi}(\xi)\hat{Q}^2((\partial^1\gamma^1 + \partial^1\gamma^1 - g^{11}\gamma \cdot \partial)\epsilon(\xi_0)\delta(\xi^2))\psi(0) \\ &\quad - \bar{\psi}(0)\hat{Q}^2((\partial^1\gamma^1 + \partial^1\gamma^1 - g^{11}\gamma \cdot \partial)\epsilon(\xi_0)\delta(\xi^2))\psi(\xi)]. \end{aligned} \quad (6)$$

The terms $\partial^1\gamma^1$ are of the size of small momenta and can be ignored in the Bjorken limit [3]. The surviving term: $\gamma \cdot \partial$ can therefore be replaced by $\gamma^+\partial^- + \gamma^-\partial^+$. Then one may evaluate the integral over $d^4\xi$. The term $\gamma^-\partial^+$ is exponentially suppressed by the large momentum q^- and is ignorable. The result is the standard parton model:

$$F_1(x) = \frac{1}{2\sqrt{2\pi}} \int d\xi^- e^{\frac{-iMx\xi^-}{\sqrt{2}}} \langle P | \psi^\dagger(\xi^-)\hat{Q}^2 P_+ \psi(0) - \psi^\dagger(0)\hat{Q}^2 P_+ \psi(\xi^-) | P \rangle|_{\xi_\perp=0, \xi^+=0}, \quad (7)$$

with $P_+ = \frac{1}{2}(1 + \alpha^3)$. Thus the parton model result emerges by taking the current commutator and keeping leading singularities [20] along the light cone [19]. In model calculations, one evaluates Eq. (7) using wave functions determined at a low momentum scale, Q_0^2 . Then QCD evolution is used to obtain distributions to compare with deep inelastic scattering data.

III. BOUND NUCLEON

The next step is to immerse the nucleon in the nucleus. There are some general features that are common to both sets of models discussed in the introduction. In each model quarks satisfy a mode equation of the general form

$$(-i\boldsymbol{\gamma} \cdot \boldsymbol{\nabla} + m(r)) q_n(\boldsymbol{\xi}) = (E_n - V^0) q_n(\boldsymbol{\xi}), \quad (8)$$

with the position dependent constituent quark mass, $m(r)$ accounting for the influence of internal binding potentials and interactions with scalar objects produced by the surrounding medium. This can be obtained by solving an in-medium gap equation for m or a self-consistency condition. The quark is also influenced by a constant vector potential V^μ , with the time component the only non-zero component for nuclear matter at rest. The time dependence of each mode is given by $e^{-iE_n\xi^0}$ with the vector potential contributing a factor $e^{-iV^0\xi^0} = e^{-i(V^+\xi^- + V^-\xi^+)}$. The field operators, now denoted as Ψ , can be expressed in terms of these mode functions that embody the influence of the medium. We now repeat the derivation of Eqs. (1)-(7), using these new field operators. The steps from Eq. (1)-Eq. (4) are as before, but the result (5) is changed to

$$\{\Psi(\xi), \bar{\Psi}(0)\} = e^{-iV \cdot \xi} \frac{1}{2\pi} \not{\partial} \epsilon(\xi_0) \delta(\xi^2). \quad (9)$$

Note the appearance of new phase factor. Using this and a similar expression for $\{\Psi(0), \bar{\Psi}(\xi)\}$ one finds an in-medium version of the structure function, \tilde{F}_1 :

$$\begin{aligned} \tilde{F}_1(x) = \frac{1}{2\sqrt{2}\pi} \int d\xi^- e^{-iP^+\xi^-} \langle P | e^{-iV^+\xi^-} \Psi^\dagger(\xi^-) \hat{Q}^2 P_+ \Psi(0) \\ - e^{iV^+\xi^-} \Psi^\dagger(0) \hat{Q}^2 P_+ \Psi(\xi^-) | P \rangle |_{\xi_\perp=0, \xi^+=0}, \end{aligned} \quad (10)$$

with P^μ the nucleon momentum. At first glance, the presence of the phase factors $e^{\mp iV^+\xi^-}$ seems to cause this result to differ substantially from that of Eq. (7). However the constant vector potential discussed above causes the mode functions to have a phase that cancels these phase factors. Thus the expression for the in-medium structure function contains no additional phase factors.

This can be expressed more formally using the techniques of Ref. [16] who observe that the quark Hamiltonian in the mean field approximation for nuclear matter at rest has the form

$$\hat{H}_q = \hat{h}_q + V_0 \hat{N}, \quad (11)$$

where \hat{h}_q is the quark Hamiltonian in the absence of the mean vector field, and $\hat{N} = \int d^3x \Psi^\dagger(x) \Psi(x)$ is the quark number operator. Ref. [16] then uses translation invariance to obtain the relation,

$$\Psi(\xi) = e^{i\hat{P}_q \cdot \xi} \Psi(0) e^{-i\hat{P}_q \cdot \xi}, \quad (12)$$

where $\hat{P}_q^\mu = (\hat{H}_q, \hat{\mathbf{P}}_q)$ the 4-momentum operator for quarks. This leads to:

$$\Psi(\xi) = e^{-iV \cdot \xi} \Psi_0(\xi). \quad (13)$$

Here Ψ_0 is the quark field in the absence of the vector potential (but influenced by the medium via scalar interactions). Using Eq. (13) in Eq. (10) allows us to obtain

$$\tilde{F}_1(x) = \frac{1}{2\sqrt{2\pi}} \int d\xi^- e^{-iP^+\xi^-} \langle P | \Psi_0^\dagger(\xi^-) \hat{Q}^2 P_+ \Psi_0(0) - \Psi_0^\dagger(0) \hat{Q}^2 P_+ \Psi_0(\xi^-) | P \rangle |_{\xi_\perp=0, \xi^+=0}. \quad (14)$$

Equation (14) tells us that the vector potential causes no shift in the argument. This follows from using the fields Ψ *everywhere* in the electromagnetic current and consistently evaluating the current commutator. Actually, the result (14) may be obtained immediately by noting that the current operator (2) expressed in terms of interacting fields Ψ can be re-expressed using (13) in terms of the quark field, Ψ_0 , obtained in the absence of the vector potential:

$$J_\mu(\xi) = \bar{\Psi}(\xi) \hat{Q} \gamma_\mu \Psi(\xi) = \bar{\Psi}_0(\xi) e^{+iV \cdot \xi} \hat{Q} \gamma_\mu e^{-iV \cdot \xi} \Psi_0(\xi) = \bar{\Psi}_0(\xi) \gamma_\mu \hat{Q} \Psi_0(\xi), \quad (15)$$

so that any exponential factors involving the vector potential are cancelled. Thus the current commutator needed to compute the structure function really only involves the fields Ψ_0 and Ψ_0^\dagger , and the vector potential causes no explicit shift in the argument.

We also note that Eq. (14) is consistent with both the baryon and momentum sum rules. The latter follows from the feature that any plus-momentum carried by the constant vector potential can be associated with the plus-momentum of the nucleon. Thus effectively, the constant vector potential carries no plus momentum [21].

Note that Eq. (14) results from using the same quark fields throughout the calculation, so that mathematical consistency is maintained. However, this consistency results from either allowing the high-momentum struck quark to interact with the same vector potential that influences quarks in the target ground state, or by ignoring the vector potential altogether. Either procedure is questionable on physical grounds.

IV. EVALUATIONS AND PHYSICS

Smith & Miller [17] and Thomas *et al.* [8, 16] each use an expression for the in-medium quark distribution function [22]:

$$\tilde{q}(x) = \frac{1}{2\sqrt{2\pi}} \int d\xi^- e^{\frac{-iP^+ x \xi^-}{\sqrt{2}}} \langle P | \Psi^\dagger(\xi^-) P_+ \Psi(0) | P \rangle |_{\xi_\perp=0, \xi^+=0}, \quad (16)$$

where P^+ is the plus component of the momentum of the bound nucleon. The phase factor $e^{-iV^+ \xi^-}$ does not appear, contradicting Eq. (10) [23]. Thomas *et al.* then exploit Eq. (13) and use consistent normalization to obtain an expression

$$q(x) = \frac{P^+}{P^+ - V^+} q_0 \left(\frac{P^+}{P^+ - V^+} x - \frac{V^+}{P^+} \right), \quad (17)$$

in which the subscript 0 refers to the absence of the vector potential. The use of Eq. (17) simplifies the evaluation of the distribution, but its use is not necessary. One could alternatively evaluate Eq. (16), and this is the procedure of Smith & Miller.

Both sets of authors avoid using the mathematically consistent, but physically questionable Eq. (14). Indeed, the models of both sets of authors are consistent with the baryon and

momentum sum rules, and produce results that are free of mathematical anomalies while achieving reasonably good descriptions of a wide variety of phenomena.

That using Eq. (16) is a better procedure than using Eq. (14) can be seen by examining Eq. (5). The presence of the phase factor in Eq. (10) arises from allowing the struck quark to feel the same mean-field vector-potential that the bound quarks experience. But such mean fields are meant only to apply to the bound particles. Indeed, the condition of asymptotic freedom mandates that the mean field should not be felt by the struck quark for very large values of Q^2 [24]. Ignoring the phase factor of Eq. (10) is one way to include the correct physics that the mean field should be energy-dependent and should disappear at large momenta. The mean-field should provide important effects for quarks in the target ground state and should vanish for high-energy quarks. Thus both sets of authors [8, 16] and [17] use a physically reasonable procedure. The rigorous task of deriving a vector potential with the ability to account for both the high and low energy limits of the quark self-energy in nuclear matter remains a task for the future.

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- [24] The structure functions are evaluated first at a relatively low-momentum transfer momentum scale ($Q_0^2 = 0.16 \text{ GeV}^2$ for Thomas *et al.* and $Q_0^2 \approx 0.36 \text{ GeV}^2$ for Smith & Miller) at which the effects of asymptotic freedom might not be dominant. Choosing to ignore the effects of the vector potential (which are not of higher twist) at the low momentum transfer scale represents part of the definition of these models.